AP Calculus BC

WS 82 Alternating Series test

1) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^2} = S$

$\cdot \lim_{n\to\infty}\frac{1}{n^2}=0$

$\cdot\ 1, \frac{1}{4}, \frac{1}{9}, \ldots$

By AST, S converges

2) $\sum_{n=1}^{\infty}(-1)^{n+1}\cdot\frac{1}{n^{3/2}} = S$

$\cdot \lim_{n\to\infty}\frac{1}{n^{3/2}} = 0$

$\cdot\ 1, \frac{1}{\sqrt{8}}, \frac{1}{\sqrt{27}}, \ldots$

S converges by AST

3) $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{n}{10}\right)^n = S$

$\cdot \lim_{n\to\infty}\left(\frac{n}{10}\right)^n = \infty \neq 0$

S diverges n^{th} term test

4) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{10^n}{n^{10}} = S$

$\cdot \lim_{n\to\infty}\frac{n^{10}}{10^n} = \infty \neq 0$

S diverges by n^{th} term test

5) $\sum_{n=2}^{\infty}(-1)^{n+1}\frac{1}{\ln n} = S$

$\cdot \lim_{n\to\infty}\frac{1}{\ln n} = 0$

$\cdot\ \frac{1}{\ln 2}, \frac{1}{\ln 3}, \frac{1}{\ln 4}, \ldots$

S converges by AST

6) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\ln n}{n} = S$

$\cdot \lim_{n\to\infty}\frac{\ln n}{n} = 0$

$\cdot\ 0, \frac{\ln 2}{2}, \frac{\ln 3}{3} \ldots$ decreasing for $n \geq e$

S converges by AST

7) $\sum_{n=2}^{\infty}(-1)^{n+1}\frac{\ln n}{\ln n^2} = S$

$\cdot \lim_{n\to\infty}\frac{\ln n}{\ln n^2} = \lim_{n\to\infty}\frac{\ln n}{2\ln n} = \frac{1}{2}$

S diverges by n^{th} term test

8) $\sum_{n=1}^{\infty}(-1)^{n}\cdot\ln\left(1+\frac{1}{n}\right) = S$

$\cdot \lim_{n\to\infty}\left[\ln\left(1+\frac{1}{n}\right)\right] = 0$

$\cdot\ \ln 2, \ln\frac{3}{2}, \ln\frac{4}{3}, \ldots$

S converges by AST

9) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sqrt{n}+1}{n+1} = S$

$\cdot \lim_{n\to\infty}\frac{\sqrt{n}+1}{n+1} = 0$

$\cdot\ 1, \frac{\sqrt{2}+1}{3}, \frac{\sqrt{3}+1}{4}, \ldots$

S converges by AST

10) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3\sqrt{n+1}}{\sqrt{n}+1} = S$

$\cdot \lim_{n\to\infty}\frac{3\sqrt{n+1}}{\sqrt{n}+1} = 3 \neq 0$

S diverges by n^{th} term test

11) $\sum_{n=1}^{\infty}(-1)^{n+1}(0.1)^n$

$S_4 = 0.1 - 0.01 + 0.001 - 0.0001$

$|S - S_4| \leq (0.1)^5 = 0.00001$

12) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(0.1)^n}{n}$

$S_4 = 0.1 - \frac{0.01}{2} + \frac{0.001}{3} - \frac{0.0001}{4}$

$|S - S_4| \leq \frac{(0.1)^5}{5} = 0.000002$

13) $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}}$

$S_4 = -1 + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}}$

$|S - S_4| \leq \frac{1}{\sqrt{5}} = 0.447$

14) $\sum_{n=1}^{\infty}\frac{(-1)^n}{1+\sqrt{n}}$

$S_4 = -\frac{1}{2} + \frac{1}{1+\sqrt{2}} - \frac{1}{1+\sqrt{3}} + \frac{1}{3}$

$|S - S_4| \leq \frac{1}{1+\sqrt{5}} = 0.309$

15) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^3+1}$

$S_4 = \frac{1}{2} - \frac{2}{9} + \frac{3}{28} - \frac{4}{65}$

$|S - S_4| \leq \frac{5}{126} = 0.0396$

16) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n!}{2^n}$

$S_4 = \frac{1}{2} - \frac{1}{2} + \frac{3}{4} - \frac{3}{2}$

$|S - S_4| \leq \frac{120}{32} = 3.75$

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